

Answering Descriptive Questions about Single Variables, Z-scores

Week 5, Psych 350 - R. Chris Fraley
<http://www.yourpersonality.net/psych350/fall2015/>

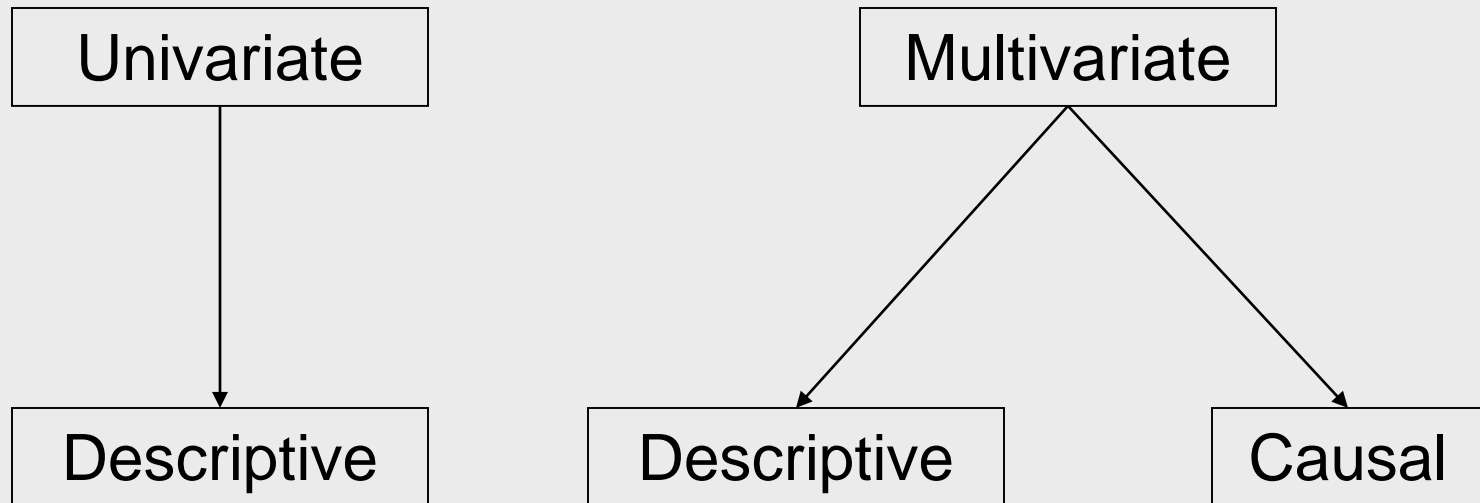
Labs This Week

- Lab on Wed: Work on importing personality survey data and organizing it in a spreadsheet that you can use effectively.
- Lab on Friday: OPTIONAL. The TAs will be there to answer questions and assist.

Different kinds of research questions

- In the next few weeks, we'll begin to talk about some of the ways that research can be designed in order to answer both basic and applied research questions.
- Some of the key questions we'll have to ask ourselves throughout this process are: (a) does this question involve one variable or more than one variable and (b) does the question concern the causal nature of the relationship between two or more variables?

Different kinds of research questions



Different kinds of research questions

- **Univariate:** questions pertaining to a single variable
 - How long are people married, on average, before they have children?
 - How many adults were sexually abused as children?
- **Descriptive** research is used to provide a systematic description of a psychological phenomenon.

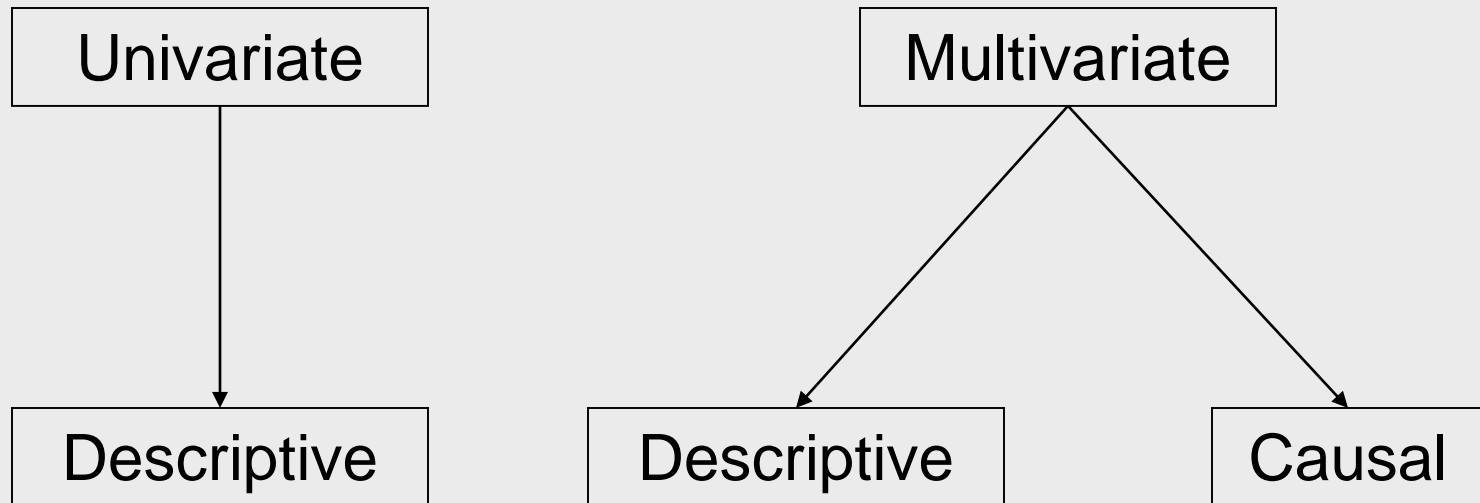
Different kinds of research questions

- **Multivariate:** questions pertaining to the relationship between two or more variables
 - How does marital satisfaction vary as a function of the length of time that a couple waits before having children?
 - Are people who were sexually abused as children more likely to be anxious, depressed, or insecure as adults?

Different kinds of research questions

- Notice that in each of these cases there is no assumption that one variable necessarily causes the other.
- In contrast, **causal** research focuses on how variables influence one another
 - Does psychotherapy help to improve peoples' well-being?
 - Does drinking coffee while studying increase test performance?

Different kinds of research questions



Univariate Descriptive Research

- The objective of univariate descriptive research is to describe a single psychological variable.

Univariate Descriptive Research

- Before we can describe the variable, we need to know whether it is *categorical* or *continuous*.
- This will impact the way we go about describing the variable.
- If the variable is categorical, all we need to do to answer the question is see what proportion of people fall into the various categories.

Categorical Variable

- Example research question: *What is the gender of students enrolled as psychology majors at UIUC?*
- We can obtain a random sample of psychology majors at UIUC.
- Measure the sex of participants (a simple self-report question should suffice)
- See what proportion of people are male vs. female.

Person	Sex
1	M
2	M
3	F
4	F
5	F
6	F
7	M
8	F
9	F

Males: 3

Females: 6

Total: 9

Males: 33% [3/9]

Females: 66% [6/9]

Continuous Variable

- When the variable is continuous it doesn't make sense to use "proportions" to answer the research question.
- Example: *How stressed is an average psychology student at UIUC?*
- To answer this question, we need to describe the *distribution* of scores.

Example

How stressed have you been in the last 2 ½ weeks?

Scale: 0 (not at all) to 10 (as stressed as possible)

4 7 7 7 8 8 7 8 9 4 7 3 6 9 10 5 7 10 6 8
7 8 7 8 7 4 5 10 10 0 9 8 3 7 9 7 9 5 8 5
0 4 6 6 7 5 3 2 8 5 10 9 10 6 4 8 8 8 4 8
7 3 7 8 8 8 7 9 7 5 6 3 4 8 7 5 7 3 3 6
5 7 5 7 8 8 7 10 5 4 3 7 6 3 9 7 8 5 7 9
9 3 1 8 6 6 4 8 5 10 4 8 10 5 5 4 9 4 7 7
7 6 6 4 4 4 9 7 10 4 7 5 10 7 9 2 7 5 9 10
3 7 2 5 9 8 10 10 6 8 3

How can we
summarize
this
information
effectively?

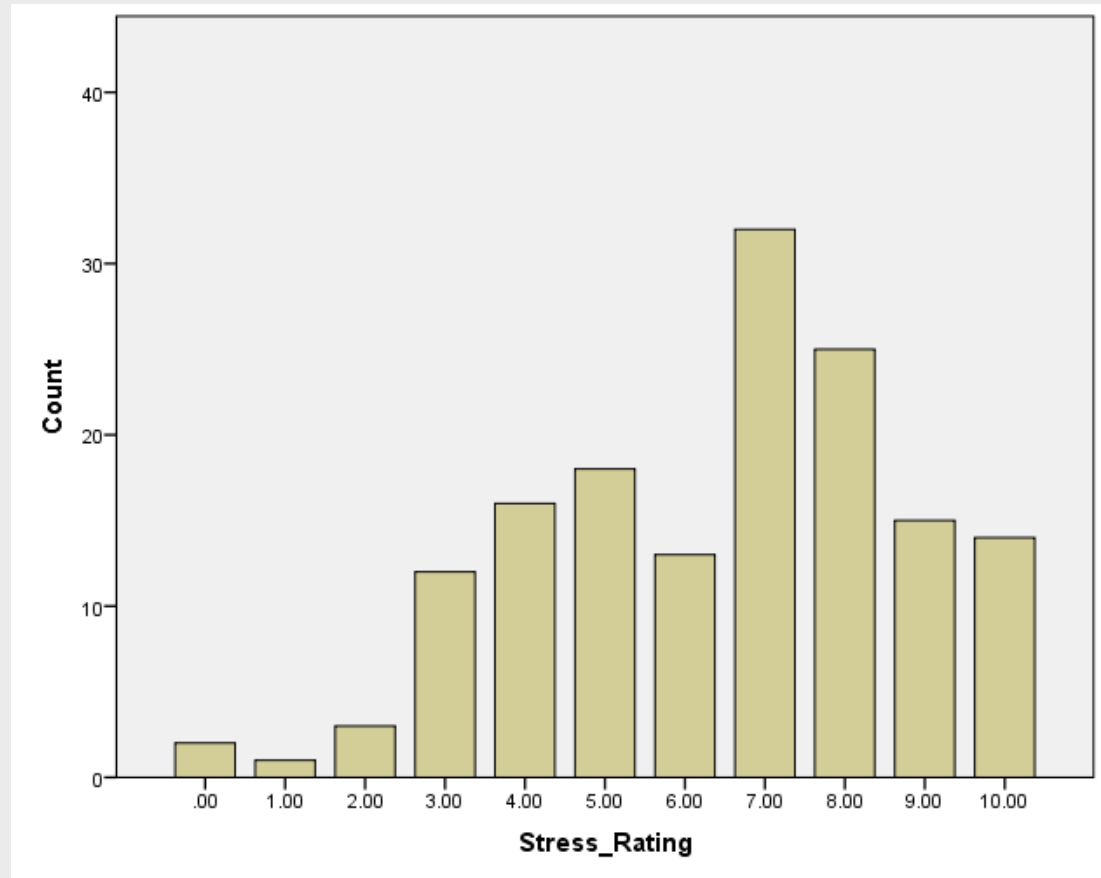
Frequency Tables

- A frequency table shows how often each value of the variable occurs

Stress rating	Frequency
10	14
9	15
8	25
7	32
6	13
5	18
4	16
3	12
2	3
1	1
0	2

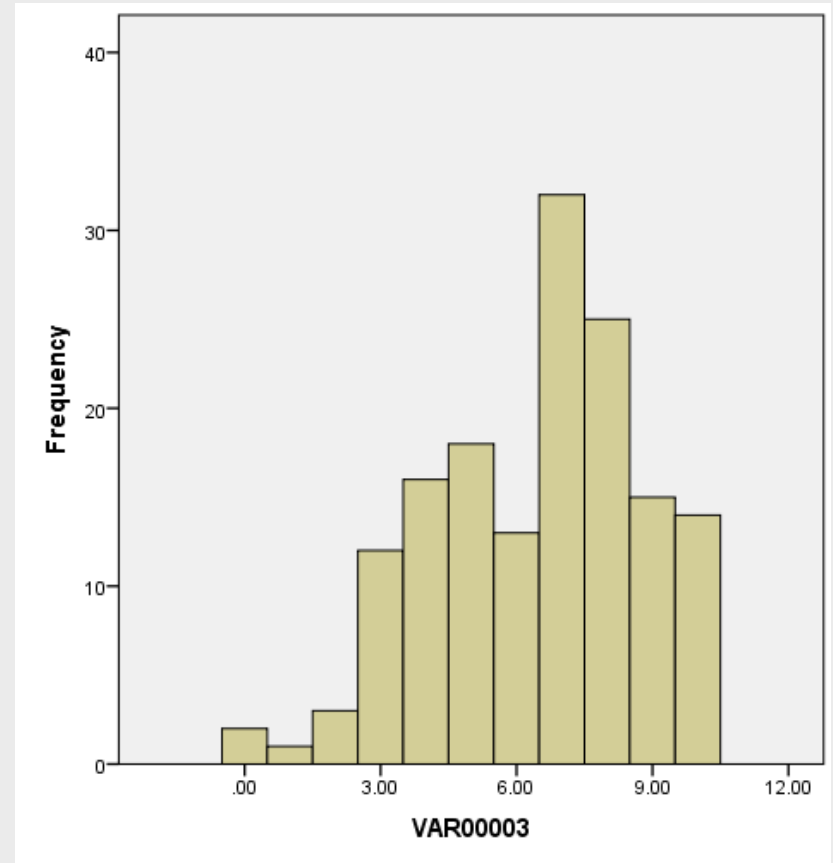
Frequency Histogram

- Bars are used to represent the frequency of each response



Histogram

- It is more typical for histograms to “bin” the scores rather than depicting the frequencies of every possible response.
- This is particularly useful when one is interested in the general distribution of scores rather than conveying information about every individual score.



Measures of Central Tendency

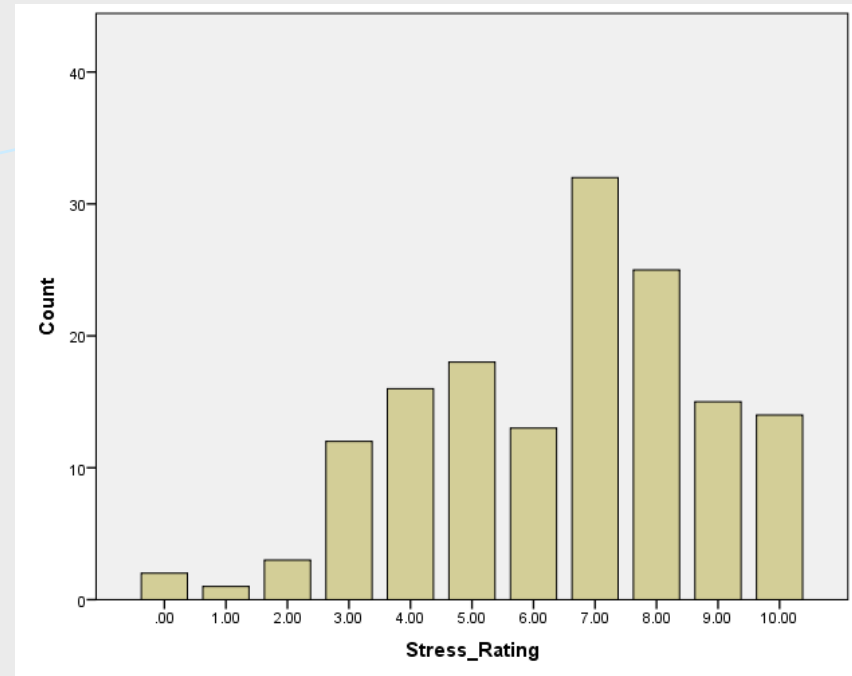
- Central tendency: most “typical” or common score
 - (a) Mode
 - (b) Median
 - (c) Mean

Measures of Central Tendency

1. Mode: most frequently occurring score

Stress	Frequency
10	14
9	15
8	25
7	32
6	13
5	18
4	16
3	12
2	3
1	1
0	2

Mode = 7



Measures of Central Tendency

2. Median: the value at which $1/2$ of the ordered scores fall above and $1/2$ of the scores fall below

1 2 3 4 5

↑
Median = 3

1 2 3 4

↑
Median = 2.5

Measures of Central Tendency

3. Mean: The “balancing point” of a set of scores; the average

x = an individual score

$$\bar{X} = M = \frac{1}{N} \sum x$$

N = the number of scores

Sigma or Σ = take the sum

- Note: Equivalent to saying “sum all the scores and divide that sum by the total number of scores”

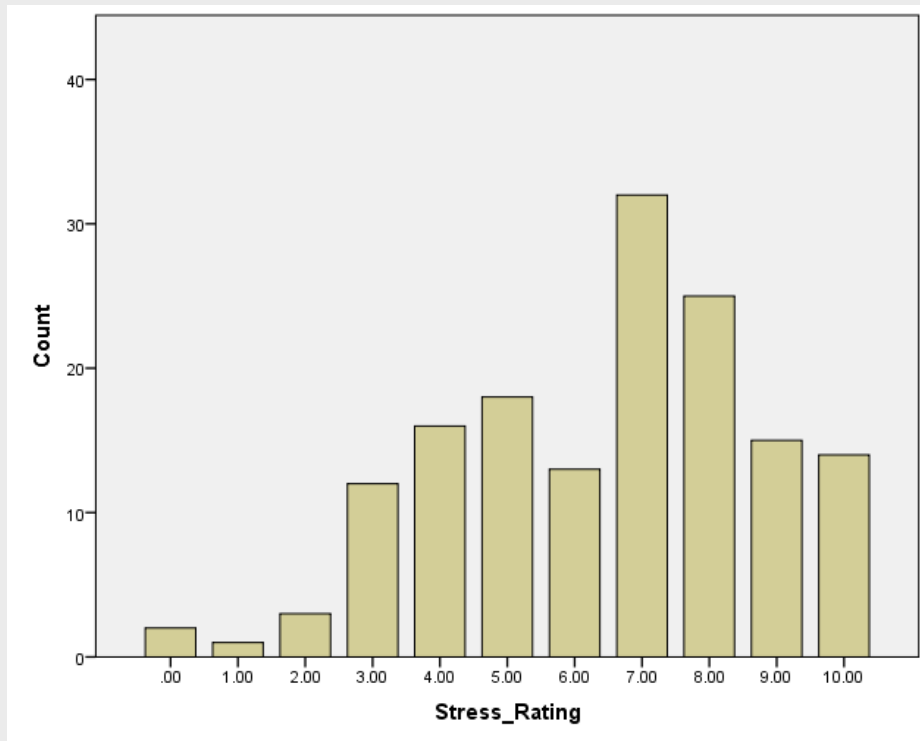
Measures of Central Tendency

Person	Score
A	1
B	2
C	2
D	3
E	3
F	3
G	3
H	4
I	4
J	5

$$\text{Mean} = (1+2+2+3+3+3+3+4+4+5)/10 = 3$$

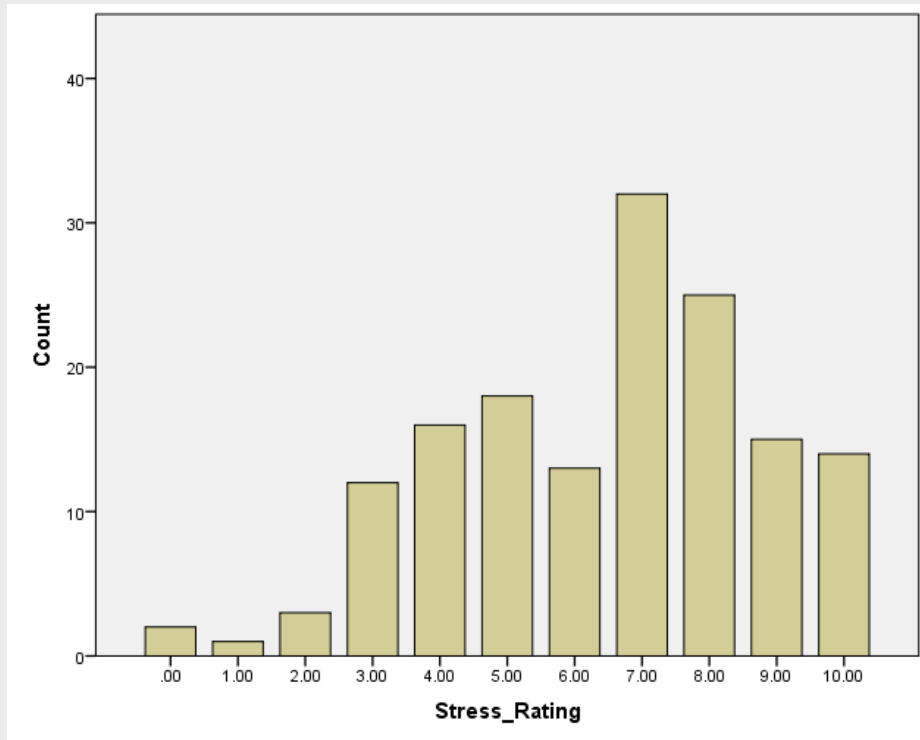
Mean

- In the stress example, the sum of all the scores is 974.
- $974 / 151 = 6.45$
- Thus, the average score is 6.45, on a 0 to 10 scale.



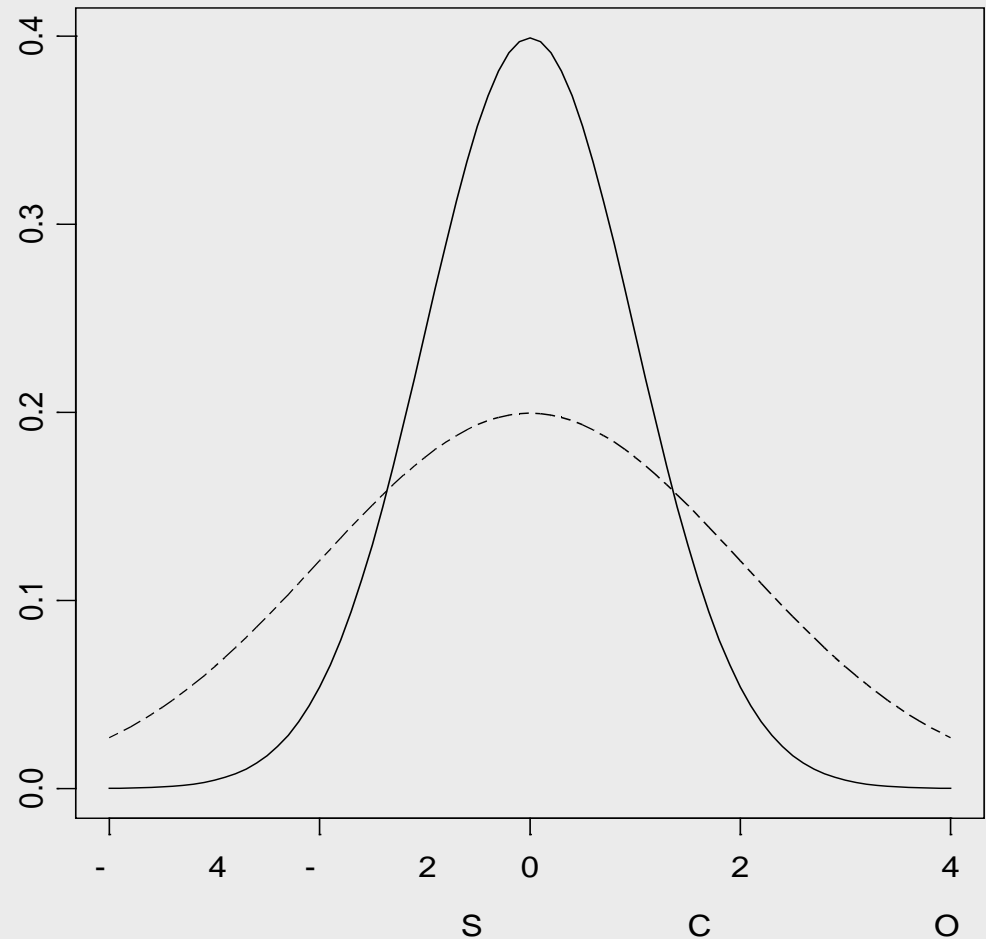
Spread

- Notice that not everyone has a score of 6.45
- Some people have very low scores (e.g., 0), and some people have very high scores (e.g., 10).
- The degree to which there is variation in the scores (i.e., people's scores differ) is referred to as the *dispersion* or *spread* of the scores.



Measures of Spread

- To illustrate the way differences in spread may look, consider this graph.
- Two sets of scores with the same mean, but different spreads.



Standard Deviation

- The most common way of quantifying dispersion is with an index called the *standard deviation*.

$$SD = \sqrt{\frac{1}{N} \sum (x - M)^2}$$

- The SD is an average, and can be interpreted as the average amount of dispersion around the mean. Larger SD = more dispersion.

Recipe for Computing the Standard Deviation

- First, find the mean of the scores. Let's call this M .
- Second, subtract *each score* from the mean. Let's call this a "mean deviation" score, which we compute for each person.
- Third, square *each* of these mean deviation scores.

These are done at the person-level

- Fourth, average these squared deviations.
- Fifth, take the square root of this average.

These are done at the group-level

Person	Score or x	(x - M)	(x - M) ²
Homer	1	(1 - 4) = -3	-3 ² = 9
Maggie	2	(2 - 4) = -2	-2 ² = 4
Lisa	2	(2 - 4) = -2	-2 ² = 4
Bart	4	(4 - 4) = 0	0 ² = 0
Marge	8	(8 - 4) = 4	4 ² = 16
Santa	7	(7 - 4) = 3	3 ² = 9

$$\frac{\sum (x - M)^2}{N} = 7$$

$$SD = \sqrt{7} = 2.64$$

$$\sum x = 24$$

$$\sum (x - M)^2 = 42$$

$$M = \frac{\sum x}{N} = 4$$

How to Verbally Summarize this Information

- In this example, we see that the average stress score is 4, on a scale ranging from 1 to 8.
- Not everyone has a score of 4, however. On average, people are 2.64 units away from the mean.

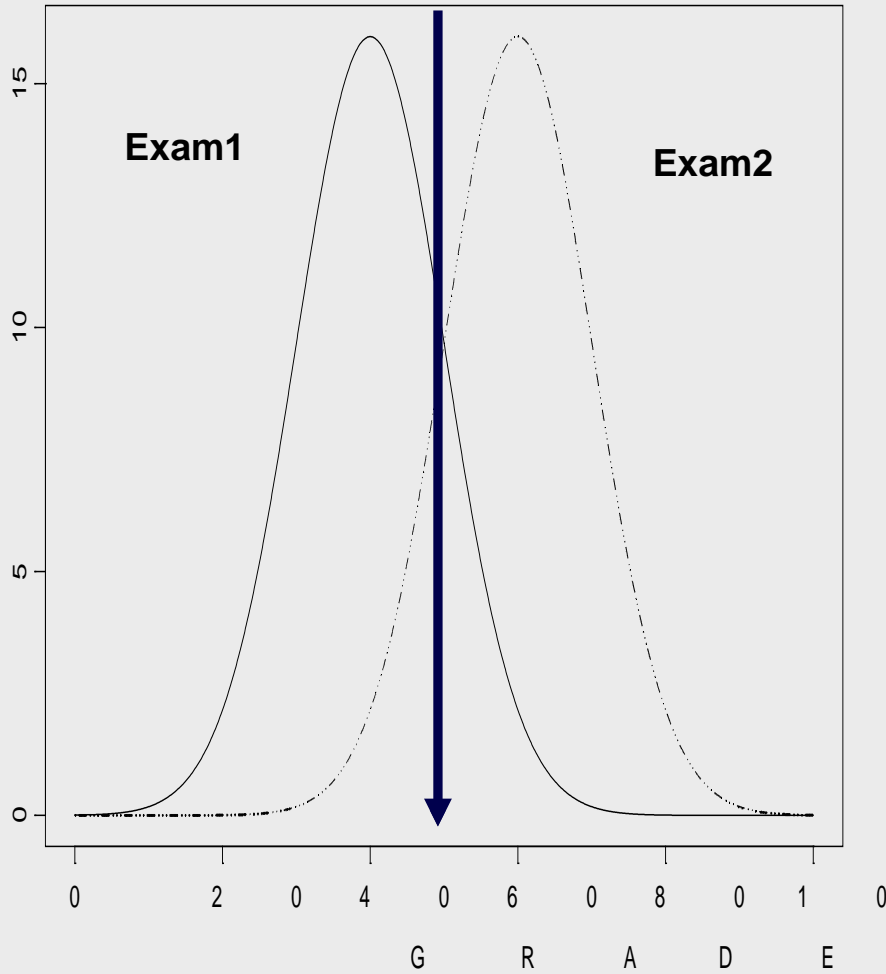
Summary

- Most descriptive questions concerning one variable can be answered pretty easily.
- If the variable is categorical,
 - determine the proportion of people in each category or level of the variable
- If the variable is continuous,
 - find the mean and standard deviation of the scores.

Making Sense of Scores

- Let's work with this first issue for a moment.
- Let's assume we have Marc's scores on his first two Psych 350 exams.
- Marc has a score of 50 on his first exam and a score of 50 on his second exam.
- On which exam did Marc do best?

Example 1

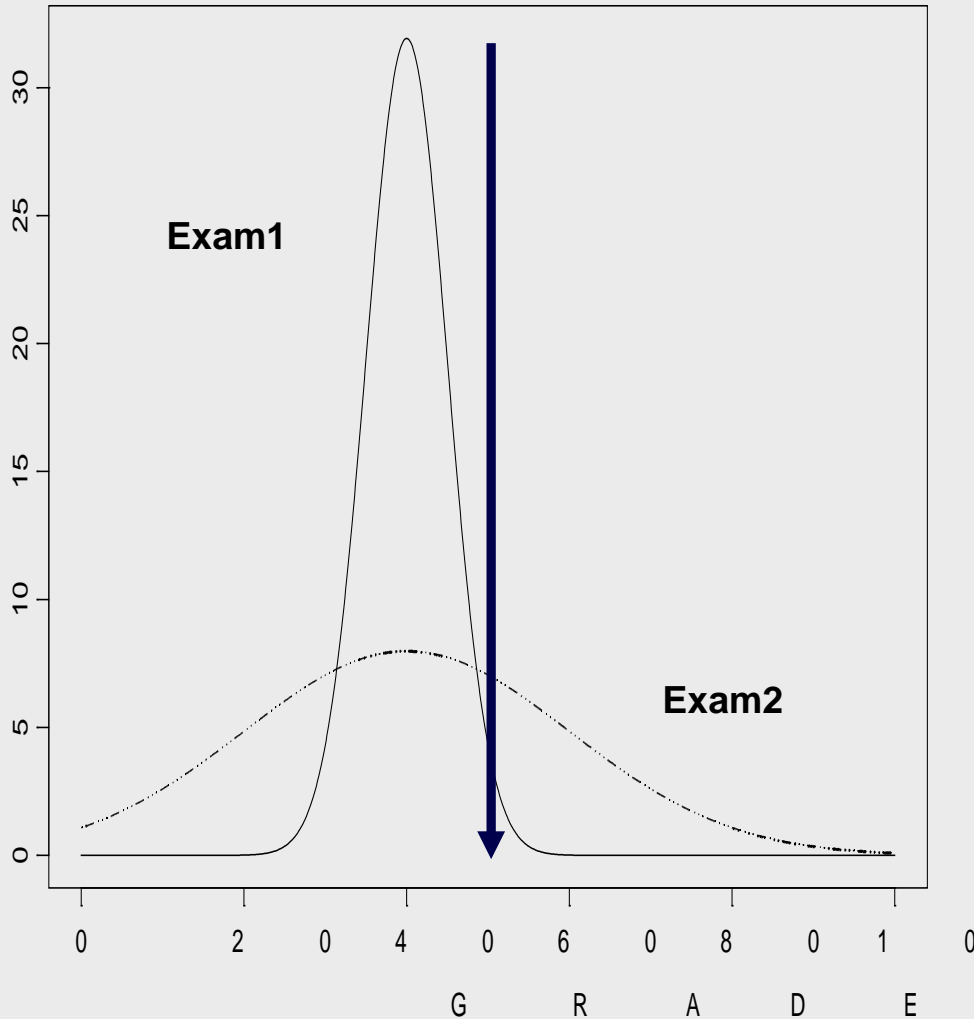


Mean Exam1 = 40

Mean Exam2 = 60

- In one case, Marc's exam score is 10 points above the mean
- In the other case, Marc's exam score is 10 points below the mean
- In an important sense, we must interpret Marc's grade relative to the *average* performance of the class

Example 2



- Both distributions have the same mean (40), but different standard deviations (10 vs. 20).
- In one case, Marc is performing better than almost 95% of the class. In the other, he is performing better than approximately 68% of the class.
- Thus, how we evaluate Marc's performance depends on how much *spread or variability* there is in the exam scores.

Standard Scores

- In short, what we would like to do is express Marc's score for any one exam with respect to (a) how far he is from the average score in the class and (b) the variability of the exam scores.
 - how far a person is from the mean:
 - $(X - M)$
 - variability in scores:
 - SD

Standard Scores

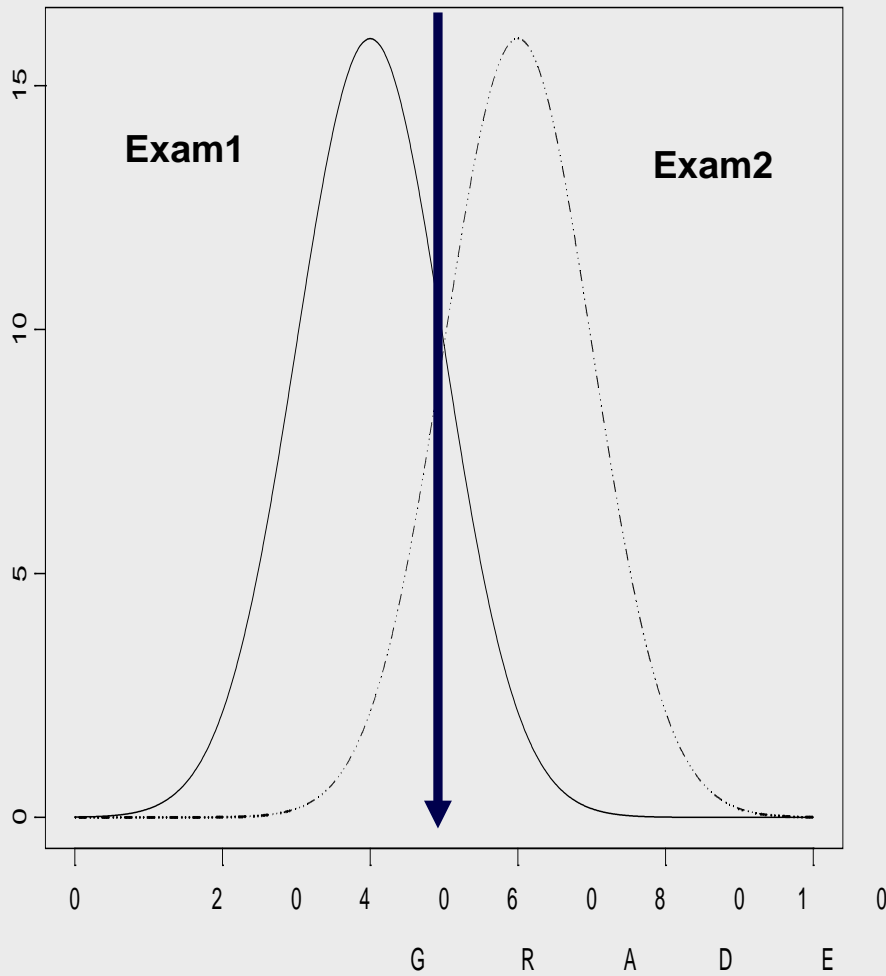
- Standardized scores, or **z-scores**, provide a way to express how far a person is from the mean, relative to the variation of the scores.

$$Z = (X - M) / SD$$

- (1) Subtract the person's score from the mean. (2) Divide that difference by the standard deviation.

*** This tells us how far a person is from the mean, in the **metric** of standard deviation units ***

Example 1



Mean Exam1 = 40

SD = 10

Mean Exam2 = 60

SD = 10

Marc's z-score on Exam1:

$$z = (50 - 40)/10 = 1$$

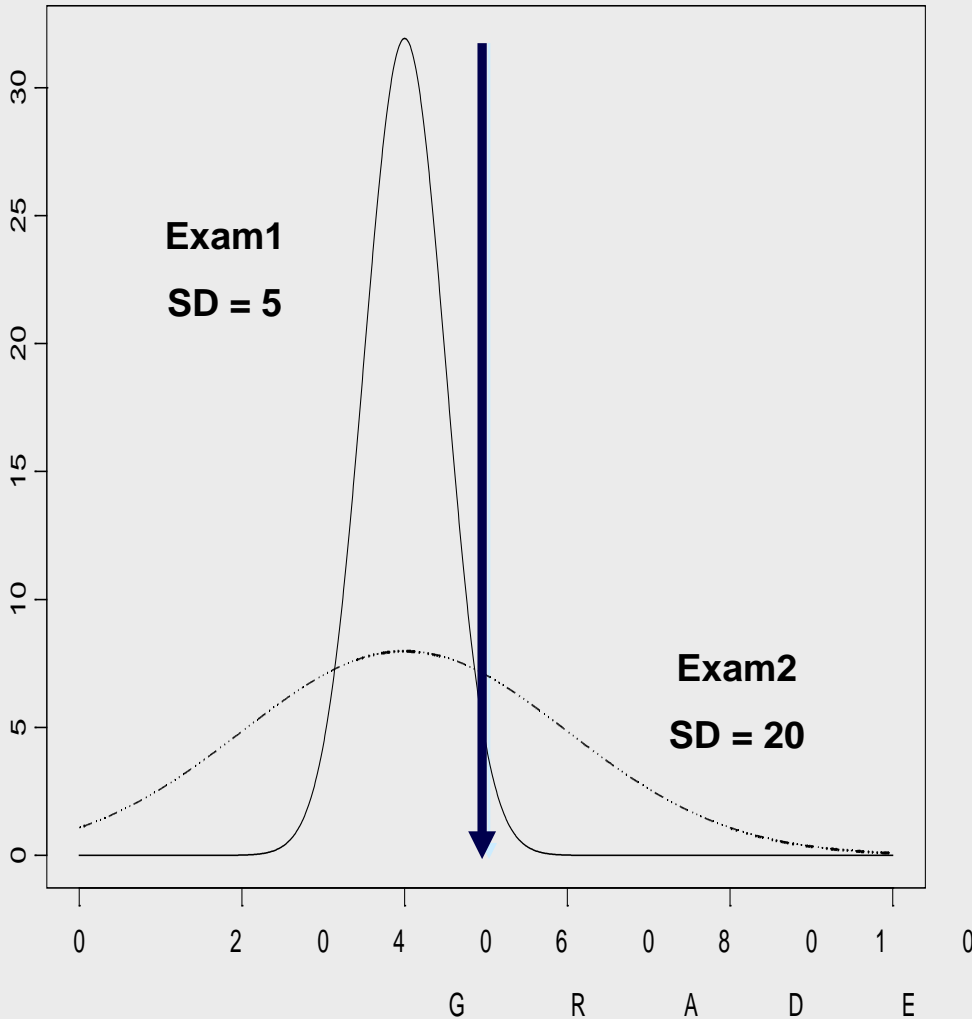
(one SD above the mean)

Marc's z-score on Exam2

$$z = (50 - 60)/10 = -1$$

(one SD below the mean)

Example 2



An example where the means are identical, but the two sets of scores have different spreads

Marc's Exam1 Z-score

$$Z = (50-40)/5 = 2$$

Marc's Exam2 Z-score

$$Z = (50-40)/20 = .5$$

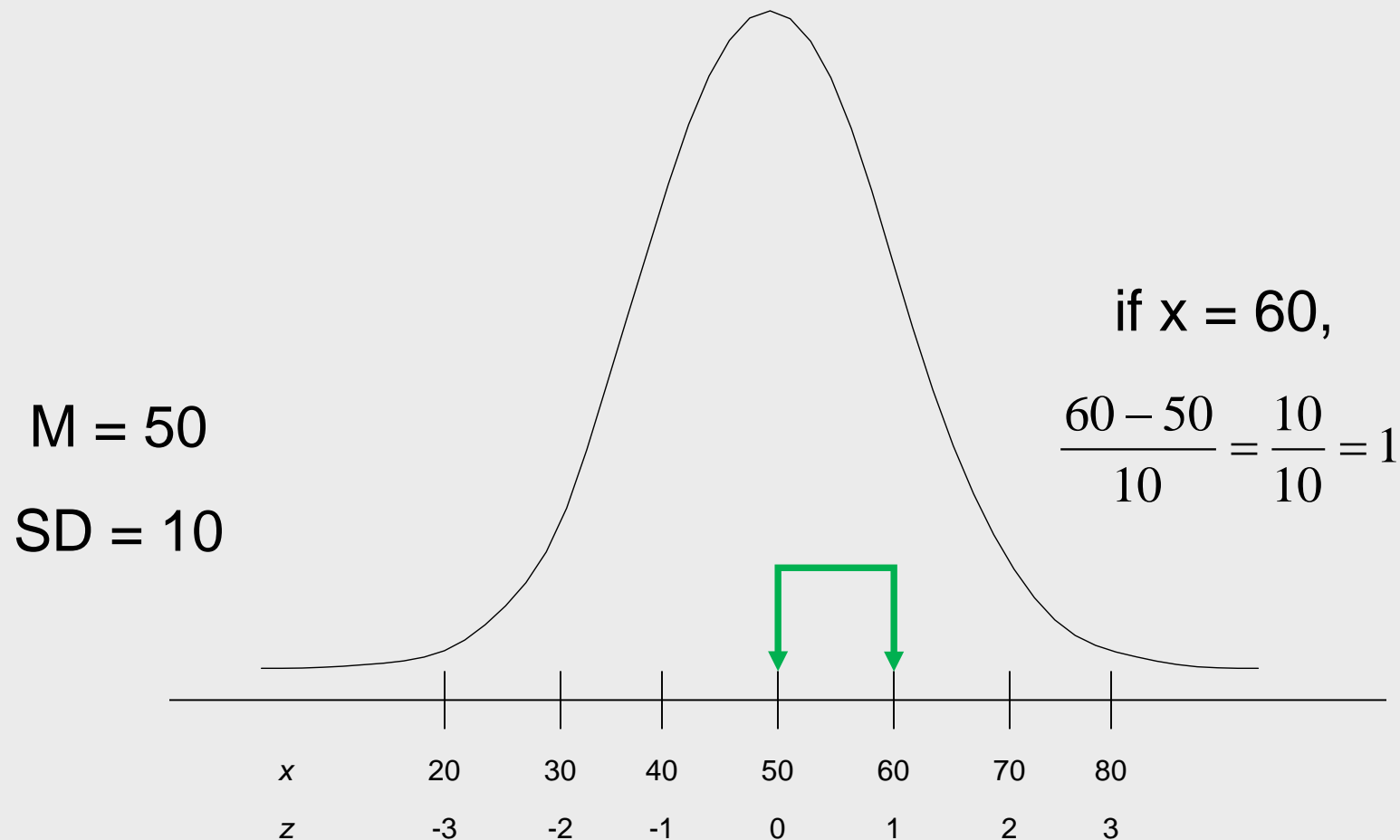
Some Useful Properties of Standard Scores

(1) The mean of a **set** of z-scores is always zero

Why? If we subtract a constant, C , from each score, the mean of the scores will be off by that amount ($M - C$). If we subtract the mean from each score, then mean will be off by an amount equal to the mean ($M - M = 0$).

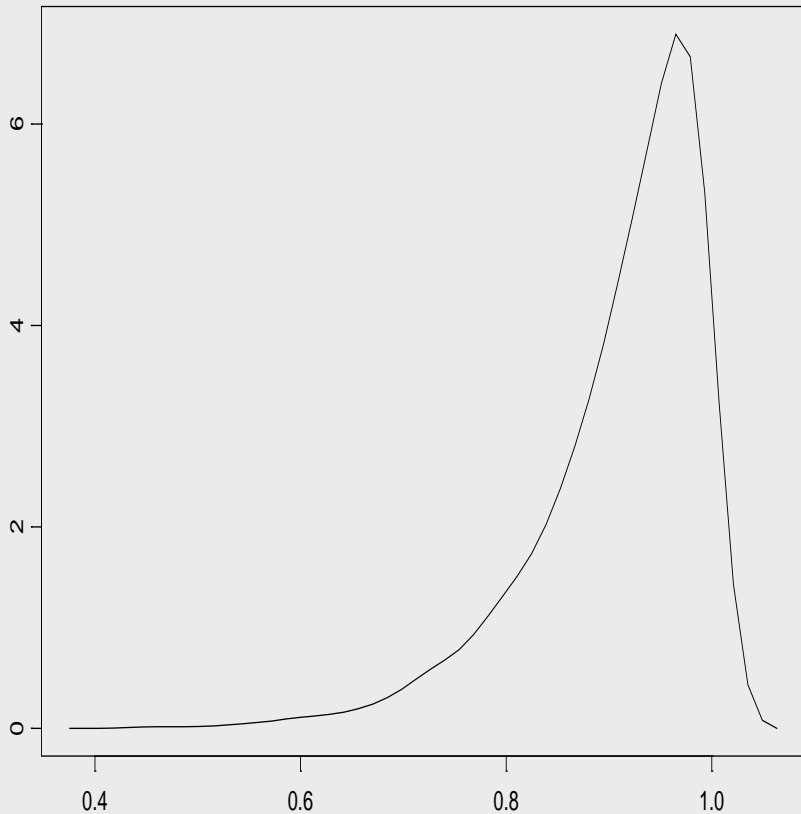
(2) The *SD* of a set of standardized scores is always 1

Why? $SD/SD = 1$; we're transforming the *metric* of the variable

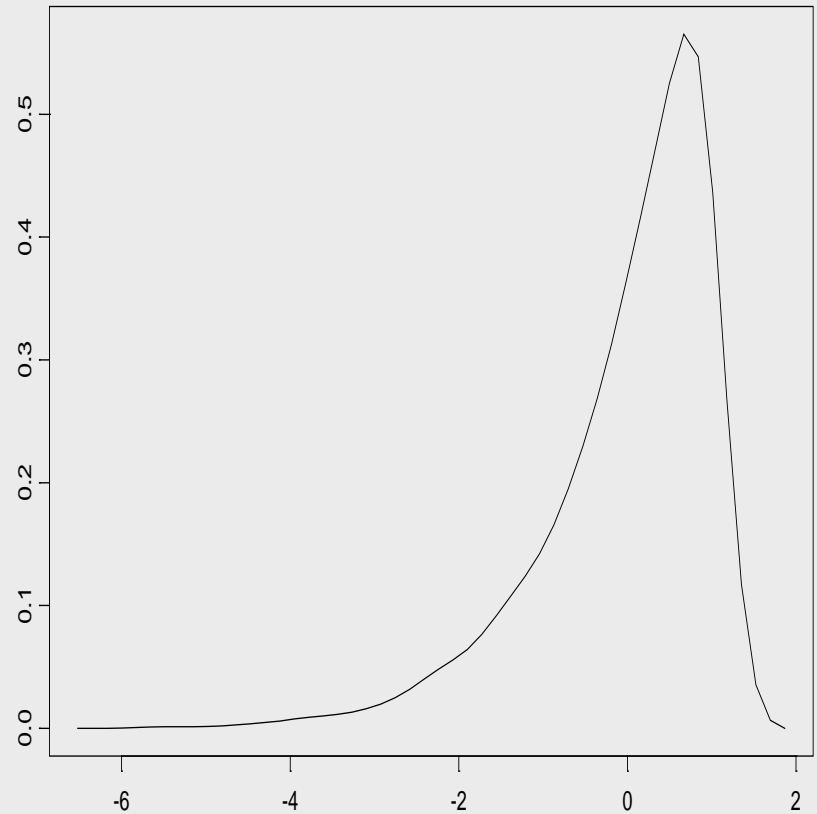


(3) The distribution of a set of standardized scores has the same shape as the unstandardized (raw) scores

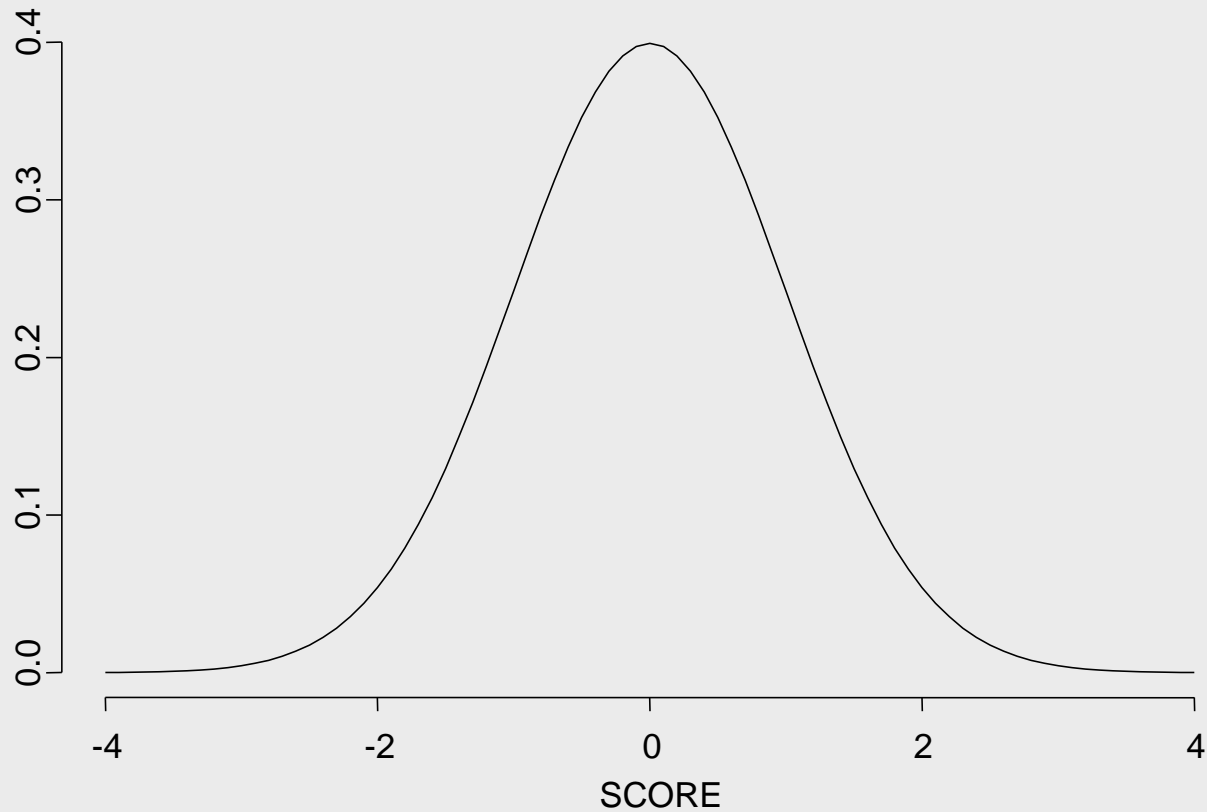
UNSTANDARDIZED



STANDARDIZED



The “normalization” (mis)interpretation

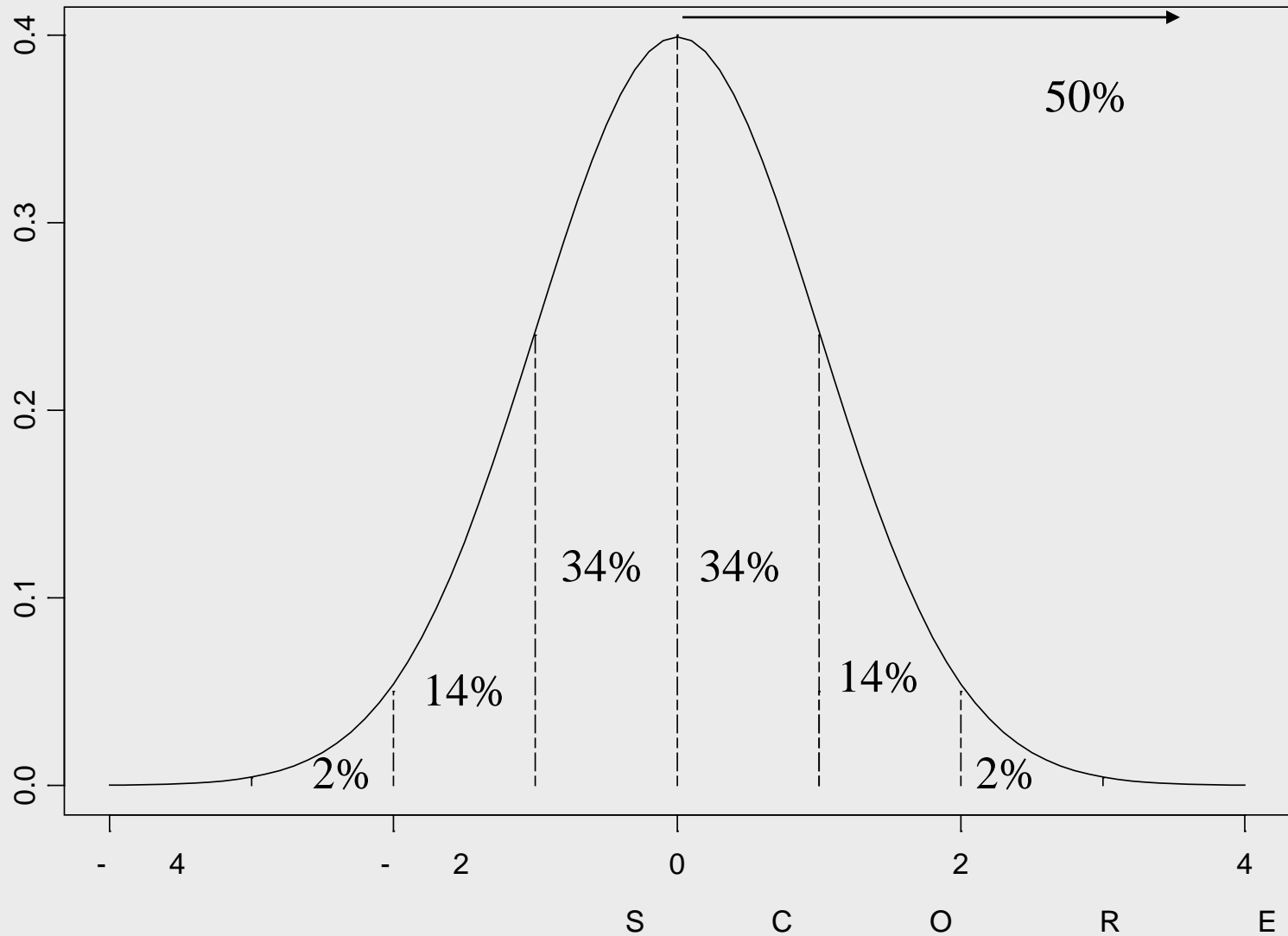


A “Normal” Distribution

Some Useful Properties of Standard Scores

- (4) Standard scores can be used to compute **centile scores**: the proportion of people with scores less than or equal to a particular score.

The area under a normal curve



Some Useful Properties of Standard Scores

- (5) Z-scores provide a way to “standardize” different metrics (i.e., metrics that differ in variation or meaning). Different variables expressed as z-scores can be interpreted on the same metric (the z-score metric). (Each score comes from a distribution with the same mean [zero] and the same standard deviation [1].)

Person	Heart Rate	Complaints	Z-score (Heart Rate)	Z-score (Complaints)	Average
A	80	2	$(80-100)/20 = -1$	$(2-2.5)/.5 = -1$	-1
B	80	3	$(80-100)/20 = -1$	$(3-2.5)/.5 = 1$	0
C	120	2	$(120-100)/20 = 1$	$(2-2.5)/.5 = -1$	0
D	120	3	$(120-100)/20 = 1$	$(3-2.5)/.5 = 1$	1
Average	100	2.5	0	0	0
SD	20	.5	1	1	1